Sequences – Write terms of a sequence and find general pattern

This section covers the following topics:

- **Define a sequence of numbers**
- **Write the first several terms of a sequence**
- **Determine a sequence from a pattern**

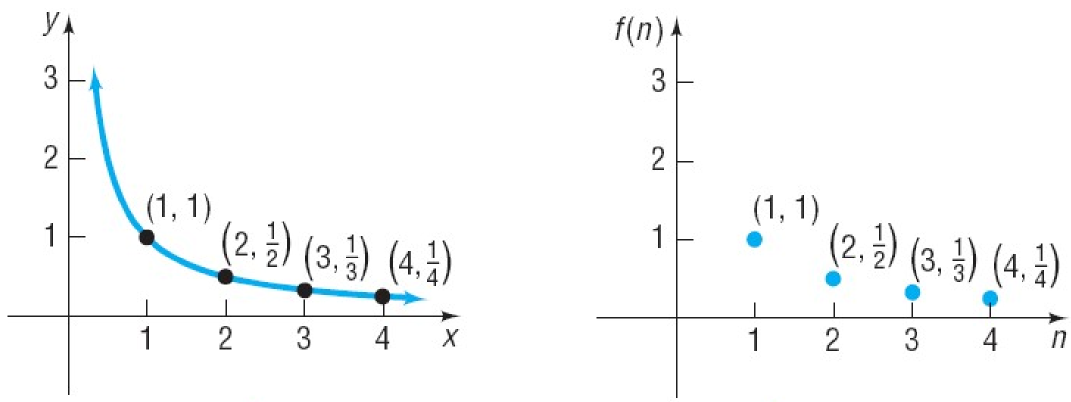
Define a Sequence of Numbers

- In the English language – a sequence can be interpreted as "a sequence of events" that are first, second, third, etcetera.

Definition: Sequence and Terms of a sequence

A **sequence** is a function whose domain is the set of positive integers.

Each number in the *ordered* list are called **terms** of a sequence.



(a) $f(x) = \frac{1}{x}, x > 0$ **(b)** $f(n) = \frac{1}{n}$, n a positive integer

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Write the first several terms of a sequence

Example 1: Represent the terms of the sequence defined by $f(n) = \frac{1}{n}$, where n is a positive integer.

$f(1) = \frac{1}{(1)} = 1$

$f(2) = \frac{1}{(2)} = \frac{1}{2}$

$f(3) = \frac{1}{3}$

$f(4) = \frac{1}{4}$

$\vdots$

$f(n) = \frac{1}{n}$

$\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots\right\}$

terms of the sequence

general or nth term of the sequence

Notation:

We often write the terms of the sequence in terms of $a_1, a_2, a_3, \ldots a_n$

In the above example, we would have the following:

$a_n = \frac{1}{n}$ $\quad a_1 = 1,\ a_2 = \frac{1}{2},\ a_3 = \frac{1}{3},\ a_4 = \frac{1}{4}, \ldots$

Definition: General Term of a sequence

The **general term** of a sequence is the formula for a given sequence.

Example 2: The sequence whose nth term is $b_n = \left(\frac{1}{2}\right)^n$ may be represented as

$b_n = \left(\frac{1}{2}\right)^n$

$b_1 = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$

$b_2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$b_3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$$\boxed{\left\{\underset{b_1}{\frac{1}{2}}, \underset{b_2}{\frac{1}{4}}, \underset{b_3}{\frac{1}{8}}, \underset{b_4}{\frac{1}{16}}, \ldots, \underset{b_n}{\left(\frac{1}{2}\right)^n}, \ldots\right\}}$$

$\parallel$

$\{b_n\}$

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Example 3: Write the first five terms of the following sequences

a. $\{s_n\} = \{n^2 + 1\}$

entire list of terms

$\{s_n\} = \{2, 5, 10, 17, 26, \ldots, n^2+1, \ldots\}$

$s_1 = (1)^2 + 1 = 2$

$s_2 = (2)^2 + 1 = 5$

$s_3 = 10$

$s_4 = 17$

$s_5 = 26$

$\{2, 5, 10, 17, 26\}$ are the 1st 5 terms

b. $\{a_n\} = \left\{\frac{2n+1}{2n}\right\}$

$a_1 = 3/2$

$a_2 = 5/4$

$a_3 = 7/6$

$a_4 = 9/8$

$a_5 = 11/10$

$\left\{\frac{3}{2}, \frac{5}{4}, \frac{7}{6}, \frac{9}{8}, \frac{11}{10}\right\}$ 1st 5 terms

c. $\{b_n\} = \left\{\frac{n^2}{2^n}\right\}$

$b_1 = \frac{1}{2}$

$b_2 = 1$

$b_3 = \frac{9}{8}$

$b_4 = 1$

$b^5 = \frac{25}{32}$

$\left\{\frac{1}{2}, 1, \frac{9}{8}, 1, \frac{25}{32}\right\}$

1st five terms

d. $\{s_n\} = \left\{(-1)^{n-1}\left(\frac{n}{2n-1}\right)\right\}$

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Determine a sequence from a pattern

Example 4: The given pattern continues. Write down the n^{th} term of the sequence

$\{a_n\}$ suggested by the pattern

a. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

$a_n = \left(\frac{1}{2}\right)^{n-1}, \quad n = 1, 2, 3, \ldots$ or $n \in \mathbb{Z}^+$

OR $a_n = \left(\frac{1}{2}\right)^{n}, \quad n = 0, 1, 2, 3, \ldots$

geometric sequence usually starts at 0

b. $1, -1, 1, -1, 1, -1, \ldots$

$a_n = (-1)^{n+1}, \quad n \in \mathbb{Z}^+$

c. $1, -2, 3, -4, 5, -6, \ldots$

$a_n = (-1)^{n+1}(n)$

d. $1, \frac{1}{2}, 3, \frac{1}{4}, 5, \frac{1}{6}, 7, \frac{1}{8}, \ldots$